Year 9 Higher:

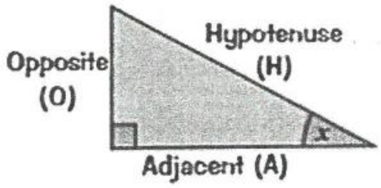
1	You can find the *nth* term of a linear sequence.
2	You can convert a fraction to a decimal. You can round to a given number of decimal places.
3	You can solve problems involving lowest common multiple (LCM).
4	You can solve problems involving factors and multiples.
5	You can solve use trigonometry to find missing sides in right-angled triangles.
6	You can use and apply Pythagoras' theorem.
7	You can solve best buy problems involving percentage changes.
8	You can calculate a percentage increase.
9	You can solve problems involving percentage increase/decrease.
10	You can find the *nth* term of a quadratic sequence.
11	You can estimate the mean from a grouped frequency table.
12	You can calculate the mean of algebraic terms.
13	You can calculate the volume of composite 3-D shapes.
14	You can calculate in standard form.

Trigonometry:

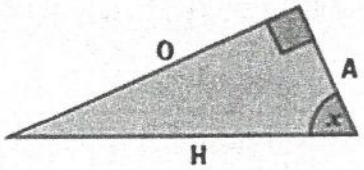
There are three basic trig formulas — each one links two sides and an angle of a right-angled triangle.

$$\text{Sin } x = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cos } x = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{Tan } x = \frac{\text{Opposite}}{\text{Adjacent}}$$

- The Hypotenuse is the LONGEST SIDE.
- The Opposite is the side OPPOSITE the angle being used (x).
- The Adjacent is the (other) side NEXT TO the angle being used.



1) Label the three sides O, A and H (Opposite, Adjacent and Hypotenuse).
2) Write down from memory 'SOH CAH TOA'.
3) Decide which two sides are involved: O,H A,H or O,A and select SOH, CAH or TOA accordingly.
4) Turn the one you choose into a FORMULA TRIANGLE:



In the formula triangles, S represents sin x, C is cos x, and T is tan x.

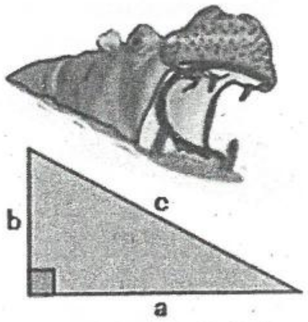
5) Cover up the thing you want to find (with your finger), and write down whatever is left showing.
6) Translate into numbers and work it out.
7) Finally, check that your answer is sensible.

If you can't make SOH CAH TOA stick, try using a mnemonic like 'Strange Orange Hamsters Creep Around Houses Tripping Over Ants'.

Pythagoras' Theorem

Pythagoras' theorem sounds hard but it's actually dead simple.
It's also dead important, so make sure you really get your teeth into it.

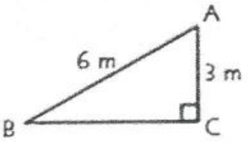
Pythagoras' Theorem — $a^2 + b^2 = c^2$ ④



$a^2 + b^2 = c^2$

1) PYTHAGORAS' THEOREM only works for RIGHT-ANGLED TRIANGLES.
2) Pythagoras uses two sides to find the third side.
3) The BASIC FORMULA for Pythagoras is $a^2 + b^2 = c^2$
4) Make sure you get the numbers in the RIGHT PLACE. c is the longest side (called the hypotenuse) and it's always opposite the right angle.
5) Always CHECK that your answer is SENSIBLE.

EXAMPLE ABC is a right-angled triangle.
AB = 6 m and AC = 3 m.
Find the exact length of BC.



It's not always c you need to find — loads of people go wrong here.

1) Write down the formula. — $a^2 + b^2 = c^2$
2) Put in the numbers. — $BC^2 + 3^2 = 6^2$
3) Rearrange the equation. — $BC^2 = 6^2 - 3^2 = 36 - 9 = 27$
4) Take square roots to find BC. — $BC = \sqrt{27} = 3\sqrt{3}$ m
5) 'Exact length' means you should give your answer as a surd — simplified if possible.

Remember to check the answer's sensible — here it's about 5.2, which is between 3 and 6, so that seems about right.

Finding the nth Term of a Quadratic Sequence ⑦

Only quadratic sequins were good enough for Cynthia

A quadratic sequence has an n^2 term — the difference between the terms changes as you go through the sequence, but the difference between the differences is the same each time.

EXAMPLE Find an expression for the nth term of the sequence that starts 4, 15, 32, 55...

$$\begin{array}{lcccc} n: & 1 & 2 & 3 & 4 \\ \text{term:} & 4 & 15 & 32 & 55 \end{array}$$

$$\begin{tikzcd}[column sep=small] 4 \arrow[r, bend right, "+11"'] & 15 \arrow[r, bend right, "+17"'] & 32 \arrow[r, bend right, "+23"'] & 55 \end{tikzcd}$$

$$\begin{tikzcd}[column sep=small] +11 \arrow[r, bend right, "+6"'] & +17 \arrow[r, bend right, "+6"'] & +23 \end{tikzcd}$$

So the expression will contain a $3n^2$ term.

term:	4	15	32	55
$3n^2$:	3	12	27	48
term − $3n^2$:	1	3	5	7

The expression for this linear sequence is $2n - 1$

So the expression for the nth term is $3n^2 + 2n - 1$

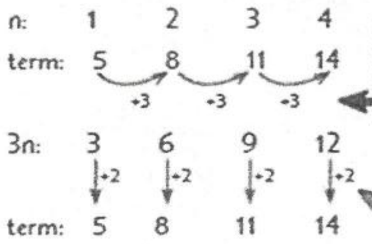
1) Find the difference between each pair of terms.
2) The difference is changing, so work out the difference between the differences.
3) Divide this value by 2 — this gives the coefficient of the n^2 term (here it's $6 \div 2 = 3$).
4) Subtract the n^2 term from each term in the sequence. This will give you a linear sequence.
5) Find the rule for the nth term of the linear sequence (see above) and add this on to the n^2 term.

Again, make sure you check your expression by putting the first few values of n back in — so n = 1 gives $3(1^2) + 2(1) - 1 = 4$, n = 2 gives $3(2^2) + 2(2) - 1 = 15$ and so on.

Finding the nth Term of a Linear Sequence

This method works for linear sequences — ones with a common difference (where the terms increase or decrease by the same amount each time). Linear sequences are also known as arithmetic sequences.

EXAMPLE Find an expression for the nth term of the sequence that starts 5, 8, 11, 14, ...



The common difference is 3, so '3n' is in the formula.

You have to + 2 to get to the term.

So the expression for the nth term is 3n + 2

1) **Find the common difference — this tells you what to multiply n by. So here, 3 gives '3n'.**
2) **Work out what to add or subtract. So for n = 1, '3n' is 3 so add 2 to get to the term (5).**
3) **Put both bits together. So you get 3n + 2.**

LCM — 'Least Common Multiple'

The SMALLEST number that will DIVIDE BY ALL the numbers in question.

If you're given two numbers and asked to find their LCM, just LIST the MULTIPLES of BOTH numbers and find the SMALLEST one that's in BOTH lists.

So, to find the LCM of 12 and 15, list their multiples (multiples of 12 = 12, 24, 36, 48, 60, 72, ... and multiples of 15 = 15, 30, 45, 60, 75, ...) and find the smallest one that's in both lists — so LCM = 60.

However, if you already know the prime factors of the numbers, you can use this method instead:

1) List all the PRIME FACTORS that appear in EITHER number.
2) If a factor appears MORE THAN ONCE in one of the numbers, list it THAT MANY TIMES.
3) MULTIPLY these together to give the LCM.

EXAMPLE $18 = 2 \times 3^2$ and $30 = 2 \times 3 \times 5$. Find the LCM of 18 and 30.

$18 = 2 \times 3 \times 3 \qquad 30 = 2 \times 3 \times 5$

So the prime factors that appear in either number are: 2, 3, 3, 5 — List 3 twice as it appears twice in 18.

$\text{LCM} = 2 \times 3 \times 3 \times 5 = 90$

The Seven Easy Rules:

Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

1) When MULTIPLYING, you ADD THE POWERS.
   e.g. $3^6 \times 3^4 = 3^{6+4} = 3^{10}$, $a^2 \times a^7 = a^{2+7} = a^9$
2) When DIVIDING, you SUBTRACT THE POWERS.
   e.g. $5^4 \div 5^2 = 5^{4-2} = 5^2$, $b^8 \div b^5 = b^{8-5} = b^3$
3) When RAISING one power to another, you MULTIPLY THEM.
   e.g. $(3^2)^4 = 3^{2\times4} = 3^8$, $(c^3)^6 = c^{3\times6} = c^{18}$
4) $x^1 = x$, ANYTHING to the POWER 1 is just ITSELF.
   e.g. $3^1 = 3$, $d \times d^3 = d^1 \times d^3 = d^{1+3} = d^4$
5) $x^0 = 1$, ANYTHING to the POWER 0 is just 1.
   e.g. $5^0 = 1$, $67^0 = 1$, $c^0 = 1$
6) $1^x = 1$, 1 TO ANY POWER is STILL JUST 1.
   e.g. $1^{23} = 1$, $1^{89} = 1$, $1^2 = 1$
7) FRACTIONS — Apply the power to both TOP and BOTTOM.
   e.g. $\left(1\frac{3}{5}\right)^3 = \left(\frac{8}{5}\right)^3 = \frac{8^3}{5^3} = \frac{512}{125}$, $\left(\frac{u}{v}\right)^5 = \frac{u^5}{v^5}$

EXAMPLE This table shows information about the weights, in kilograms, of 60 school children.

a) Write down the modal class.
b) Write down the class containing the median.
c) Calculate an estimate for the mean weight.
d) Estimate the range of weights.

Weight (w kg)	Frequency
$30 < w \le 40$	8
$40 < w \le 50$	16
$50 < w \le 60$	18
$60 < w \le 70$	12
$70 < w \le 80$	6

a) **The modal class is the one with the highest frequency.**

Modal class is $50 < w \le 60$

b) **Work out the position of the median, then count through the 2nd column.**

The median is in position $(n + 1) \div 2 = (60 + 1) \div 2 = 30.5$, halfway between the 30th and 31st values. Both these values are in the third class, so the class containing the median is $50 < w \le 60$.

c) **Add extra columns for 'mid-interval value' and 'frequency × mid-interval value'. Add up the values in the 4th column to estimate the total weight of the 60 children.**

Weight (w kg)	Frequency (f)	Mid-interval value (x)	fx
$30 < w \le 40$	8	35	280
$40 < w \le 50$	16	45	720
$50 < w \le 60$	18	55	990
$60 < w \le 70$	12	65	780
$70 < w \le 80$	6	75	450
Total	60	—	3220

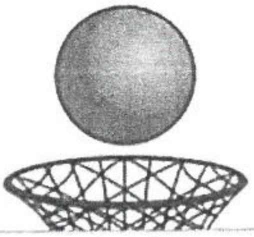
Don't add up the mid-interval values.

$$\text{Mean} \approx \frac{\text{total weight}}{\text{number of children}}$$

total weight ← 4th column total
number of children ← 2nd column total

$$= \frac{3220}{60} = 53.7 \text{ kg (3 s.f.)}$$

Volumes of Spheres

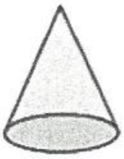


VOLUME OF SPHERE = $\frac{4}{3}\pi r^3$

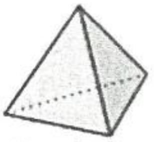
A hemisphere is half a sphere. So the volume of a hemisphere is just half the volume of a full sphere, $V = \frac{2}{3}\pi r^3$.

Volumes of Pyramids and Cones

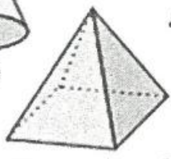
A pyramid is a shape that goes from a flat base up to a point at the top. Its base can be any shape at all. If the base is a circle then it's called a cone (rather than a circular pyramid).



Cone



Tetrahedron



Square-based pyramid

VOLUME OF PYRAMID = $\frac{1}{3}$ × BASE AREA × VERTICAL HEIGHT

VOLUME OF CONE = $\frac{1}{3} \times \pi r^2 \times h_v$

Make sure you use the vertical (perpendicular) height in these formulas — don't get confused with the slant height, which you used to find the surface area of a cone.

Standard Form

Calculations with Standard Form ⑤

These are really popular exam questions — you might be asked to add, subtract, multiply or divide using numbers in standard form without using a calculator.

Multiplying and Dividing — not too bad

1) Rearrange to put the front numbers and the powers of 10 together.
2) Multiply or divide the front numbers, and use the power rules (see p.17) to multiply or divide the powers of 10.
3) Make sure your answer is still in standard form.

EXAMPLES

1. Find $(2 \times 10^3) \times (6.75 \times 10^5)$ without using a calculator. Give your answer in standard form.

$(2 \times 10^3) \times (6.75 \times 10^5)$

Multiply front numbers and powers separately

$= (2 \times 6.75) \times (10^3 \times 10^5)$

$= 13.5 \times 10^{3+5}$ Add the powers (see p.17)

$= 13.5 \times 10^8$

Not in standard form — convert it

$= 1.35 \times 10 \times 10^8$

$= 1.35 \times 10^9$

2. Calculate $240\,000 \div (4.8 \times 10^{10})$ without using a calculator. Give your answer in standard form.

$240\,000 \div (4.8 \times 10^{10})$

Convert 240 000 to standard form

$= \dfrac{2.4 \times 10^5}{4.8 \times 10^{10}} = \dfrac{2.4}{4.8} \times \dfrac{10^5}{10^{10}}$

Divide front numbers and powers separately

$= 0.5 \times 10^{5-10}$ Subtract the powers (see p.17)

$= 0.5 \times 10^{-5}$

Not in standard form — convert it

$= 5 \times 10^{-1} \times 10^{-5}$

$= 5 \times 10^{-6}$